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THERMAL CAPACITY OF A WALL WITH A TOROIDAL PLASMA INCLUSION[1]

R. Krichel, H. Druxes and G. Schmitz[2]

ABSTRACT. A model theory is demonstrated to determine the load capacity of a wall enclosing an electric arc. The theory is based on exact numerical computations of the temperature distribution inside the wall of a toroidal arc tube. It is shown and explained that the thermal load capacity increases with the radius of curvature with fixed tube radius and decreases with tube radius at a fixed radius of curvature.

1. Introduction

An electric arc at a pressure of 1 atmosphere in a toroidal gas-impermeable /13* tube with circular cross section is the object of the following study, which deals with its heat transmission in the wall as well as the maximum thermal resistance of the wall. The influenceability model from plasma physics [1-4] is used for the theoretical study of the phenomenon in the arc column as well as the arc wall. In conjunction with the special boundary conditions for the torus problem, the equations in this theory offer not only relationships between thermal and electrical arc parameters [2], but also offer the possibility of expressing the maximum wall stress as a function of tube parameters and the thermal properties of the wall material. In this connection, it is always assumed that the ratio of the arc cross sectional radius to the torus radius can be very small. In such a "slightly curved arc" the asymmetry of the /14 eigenmagnetic forces is small, so that the enthalpy product term is not involved in the power balance.

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[1]Excerpt from dissertation of Mr. Krichel submitted for the degree of Doctor of Natural Sciences at the Mathematics and Natural Science Faculty of the Rhineland-Westphalian Technical College, Aachen. Date of graduation: 10 July 1970.

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*Numbers in the margin indicate pagination in the foreign text.

2. Torus Arc With Cooling Mechanism

To study the problem at hand, we shall proceed on the basis of a torus with circular cross section, into which a narrow insulator with electrodes mounted on both sides has been inserted. The arc burns in the gas-filled torus from one electrode to the other. Figure 1 shows a torus arc element with radius of curvature R, with the skeleton and energy fluxes inside and outside the wall in cross section, with a being the internal radius of the tube and d the wall thickness. The outer skin of the arc is maintained by a liquid or gaseous coolant at an external temperature T_k remains constant with time on the average. The temperature T_w on the inner wall of the arc is determined by the outer wall temperature and the geometrical parameter. The total current strength, maximum arc temperature or electrical power per unit length can also be used as further determining parameters.

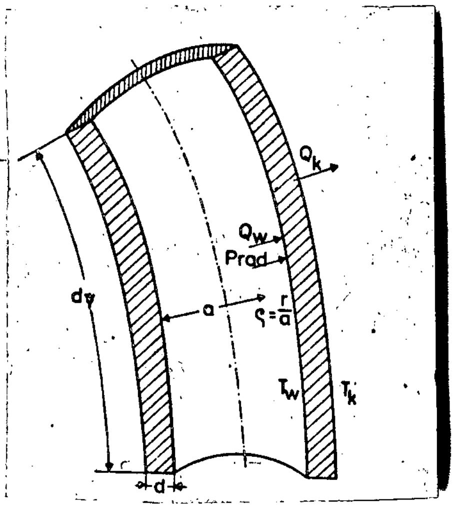

Figure 1. Torus Arc Geometry With Skeleton and Energy Fluxes In the Wall.

The coordinates employed are the polar angle ψ in the direction of the skeleton, the normalized tube radius $\rho = r/a$ as well as the azimuth angle in the tube-cross sectional area φ.

In a torus arc, as in a straight cylindrical arc, 2 different cooling mechanisms may be involved [5]. The coolant, e.g., nitrogen gas, may be /15 blown through the wall radially from outside. This case, in which thermal convection as well as radiation and heat conduction play an important role in the cooling process, will not be discussed in greater detail here, since we are studying a torus without internal influx of gas. The other case is the one without influx, in which cooling proceeds solely on the basis of thermal conduction and radiation of heat.

The temperature created in the outer skin depends on the one hand on the thermal and radiated power given off at the wall and on the other hand on the thermal transportability of the coolant. The internal wall temperature, due to the danger of vaporization and melting of the wall material, has imposed upon it an upper limit which cannot be exceeded.

Under the assumption that the internal wall temperature is within the abovementioned boundary temperature and enthalpy production is stationary in the arc, the power conversion can be described by the following equations:

$$P_{el} = Q_w + P_{rad}$$

$$Q_w + P_{rad} = Q_k.$$

The first equation expresses the fact that the electrical power introduced from outside per unit length P_{el} is converted to the heat flux per unit length Q_w and the radiated power per unit length P_{rad} at the wall (Figure 1). The second equation describes the special nature of the cooling mechanism employed. All of the energy absorbed by the inner wall is transmitted in the outer wall to the coolant by the heat flux per unit length Q_k.

3. Model Theory For Determination of the Maximum Wall Strength

As has already been discussed in considerable detail in [2], the power balance in the arc wall has the form

$$\frac{\cos\varphi}{\frac{R}{a}+\rho\cos\varphi}\frac{\partial S}{\partial\rho}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial S}{\partial\rho}\right) -\frac{\sin\varphi}{\frac{R}{a}+\rho\cos\varphi}\frac{1}{\rho}\frac{\partial S}{\partial\varphi}+\frac{1}{\rho^2}\frac{\partial^2 S}{\partial\varphi^2}=0. \qquad (3.1)$$

This equation differs from that of the arc only in that the production term of the electrical energy as well as the radiation term are absent. The power /16 balance cannot be solved analytically. For each solid but arbitrary angle and with disregard of the azimuthal dependence of the heat flux potential for this angle, however, there is an equation which may be solved analytically and (as a comparison with the exact numerical solution of (3.1) for this angle shows) indicates relatively well the curve of the heat flux potential in this point in the wall. We are selecting the angle $\varphi = \pi$ especially because in this case, due to the curvature of the arc, there is maximum thermal stress. The heat flux potential in the wall of the torus with a radius of curvature R, tube radius a, wall thickness d and energy flux density $j_E(\rho, \varphi)$ in the inner wall and the heat flux potential S_k in the outer skin will then be determined with $\varphi = \pi$ by the following:

$$\frac{S(\rho,\pi)-S_k}{a j_E(1,\pi)} = \frac{1}{1+\dfrac{a}{R\cdot\left(1-\dfrac{a}{R}\right)}} \ln \frac{\left(1-\dfrac{a}{R}\rho\right)\left(1+\dfrac{d}{a}\right)}{\left(1-\dfrac{a}{R}\left(1+\dfrac{d}{a}\right)\right)\rho} \qquad (3.2)$$

This formula, for large radii of curvature R, is converted to the already familiar logarithmic distribution of the cylindrically symmetrical, wall-stabilized. flow-free arc with constant wall temperature [4]. The degree to which this analytical distribution is in good agreement with that which was calculated numerically is evident from Figure 2, where the same numerical wall distribution as in Figure 3 for $\varphi = \pi$ is compared on an enlarged scale with /17
that determined according to (3.2). The numerically calculated distribution is located above the model solution due to consideration of azimuthal thermal conduction.

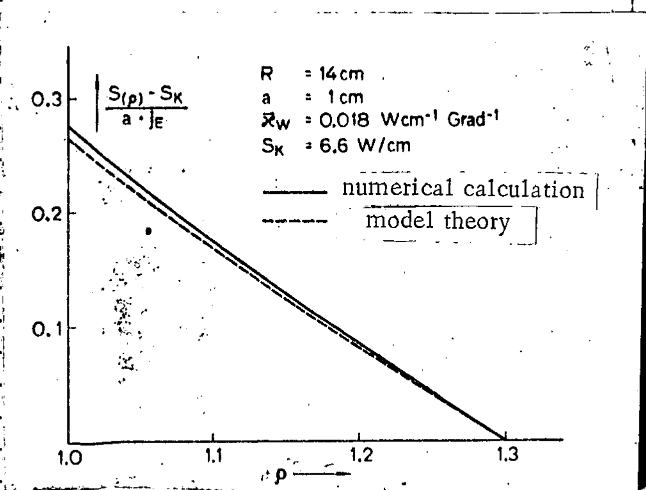

Figure 2. Comparison of Numerically Normalized Distribution of Heat Flux Potential and the Value Obtained By Means of A Model Theory In A Quartz Wall With $\varphi = \pi$.

Due to the good agreement between the numerical distribution and the model theory, it is permissible to draw further important conclusions from the model theory.

Relationship (3.2) now offers the possibility of making a statement regarding the maximum thermal stress on the wall. By this we mean the value of the maximum energy flux density $j_{E\,max}(1, \varphi)$ in the inner wall of the torus with $\varphi = \pi$. Due to the azimuthal dependence on φ caused by the curvature, in this instance we cannot use the converted electrical power per arc length P_{el} as a descriptive value for the wall strength. P_{el}, as an integral value, is only equivalent to $j_{E\,max}$ in a straight cylindrical arc with constant external wall temperature, since in this case $j_{E\,max}$ is independent of the azimuthal angle φ due to rotational symmetry.

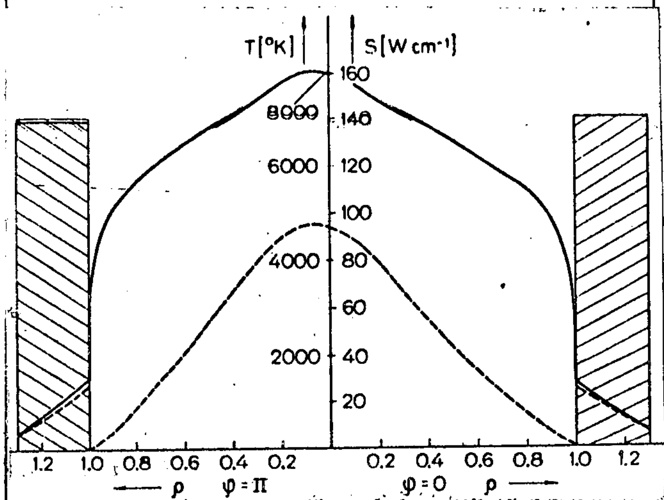

Figure 3. Temperature Distribution (Solid Curve) and Heat Flux Potential (Dashed Curve) In A Torus Arc With Quartz Wall Along Axis C = 0, π (R = 14 cm, a = 1 cm, $\bar{k}$ = 0,008 W_{cm}^{-1} $degree^{-1}$, T_A = 8,000°K, b = 138, 415, $S(1, \varphi) = 0.1867 \exp(\varphi^2/b)$ W_{cm}-1).

If the heat flux potential at the melting point of the specific wall material is introduced into (3.2) for S on the inner wall, the solution for j_E will give the maximum permissible energy flux density in the wall at the point /18
$\varphi = \pi$:

$$j_{E_{max}} = \left(1 + \frac{\frac{a}{R}}{1 - \frac{a}{R}}\right) \frac{\bar{k}(T_s - T_k)}{a \cdot \ln\left(\frac{\left(1 - \frac{a}{R}\right)\left(1 + \frac{d}{a}\right)}{1 - \frac{a}{R}\left(1 + \frac{d}{a}\right)}\right)} \tag{3.3}$$

In this equation the difference between the heat flux potential at the melting point and at the temperature of the coolant $S_s(1, \varphi) - S_k$ is given in good approximation by $\bar{k}(T_s - T_k)$. Here $\bar{k}$ is the thermal conductivity of the wall material averaged over the temperature. The thermal wall strength at $\varphi = \pi$ is therefore dependent on the wall thickness d, the difference between the melting point T_s of the wall material and the coolant temperature T_k, the torus radius a and the radius /19
of curvature R. Since the point $\varphi = \pi$ must withstand the maximum heat stress due to the displacement of the arc, (3.3) allows determination of the maximum wall strength in general. For R tending toward infinity, this relationship becomes that of the straight cylindrical arc.

TABLE. MELTING POINT T_s, AVERAGE THERMAL CONDUCTIVITY AND NORMALIZED THERMAL STRESS $\bar{\kappa}(T_s - T_k)$ FOR VARIOUS WALL MATERIALS. THE VALUES a) CORRESPOND TO A TEMPERATURE OF T = 300°K, THE VALUES, b) CORRESPOND TO A TEMPERATURE OF T = 700°K, c) CORRESPONDS TO A TEMPERATURE OF 2,400°K.

	$\kappa\left[\frac{\text{cal}}{\text{cm}\cdot\text{s}\cdot\text{grad}}\right]$	T_s [°C]	$\dfrac{j_{E\max}\cdot a\cdot \ln\dfrac{\left(1-\frac{a}{R}\right)\left(1+\frac{d}{a}\right)}{1-\frac{a}{R}\left(1+\frac{d}{a}\right)}}{1+\dfrac{a/R}{1-\frac{a}{R}}}=\kappa(T_s-T_k)\left[\frac{\text{kW}}{\text{cm}}\right]$	
			$T_k = 300$ °K	$T_k = 700$ °K
Metal				
Cu (99,9%)	a) 0,951	1083	a) 4,203	—
Ag (99,9%)	a) 0,999	961,3	a) 3,907	—
Al	a) 0,569	659	a) 1,505	—
Fe	a) 0,173	1536	a) 1,092	—
Ni	a) 0,161	1455	a) 0,962	—
	b) 0,125		b) 0,747	b) 0,538
W	b) 0,3	3390	b) 4,223	b) 3,720
	c) 0,35		c) 4,926	c) 4,341
Fe	a) 0,174	1535	a) 1,098	—
(Armco)	b) 0,117		b) 0,738	b) 0,543
Nonmetal				
C	a) 0,403	3800	a) 6,364	—
	b) 0,242		b) 3,822	b) 3,416
Quartz glass	a) 0,003	1470	a) 0,018	—
	b) 0,004		b) 0,024	b) 0,017

As we can see from (3.3), the stress will increase as the average thermal conductivity of the wall k and the temperature differential between the melting point and the coolant temperature increase and the wall thickness decreases. These findings become immediately understandable if we recall that the wall is able to give off per unit time an amount of heat which increases with increasing average thermal conductivity and increasing temperature gradient across the wall.

In order to study the dependents of the maximum energy flux across the wall on the curvature, we have plotted $j_{E\max}$ as a parameter in Figure 4 for a quartz wall as a function of the radius of curvature for 3 different tube radii. /20

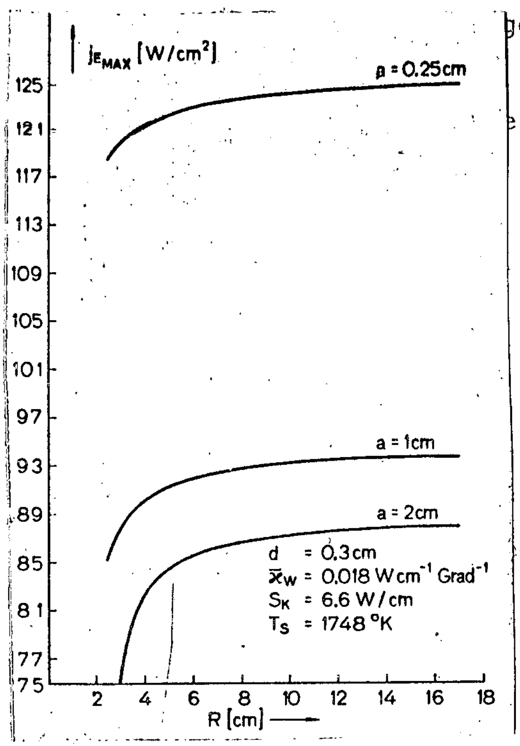

Figure 4. The Maximum Possible Energy Flux Density In the Wall In the Case of Quartz As A Function of Radius of Curvature For 3 Different Tube Radii.

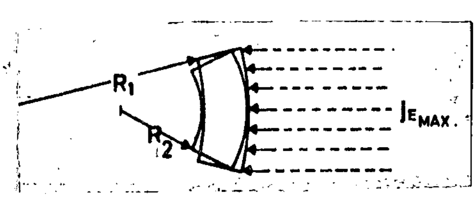

Figure 5. Diagram Showing the Dependence of the Maximum Energy Flux Density At $\varphi = \pi$ On the Wall Curvature.

All of the curves show unquestionably that the maximum wall stress curvature with decreasing curvature, i.e., with increasing radius of curvature. This behavior is explained by Figure 5. The energy flux density which strikes the inner wall of the torus must be removed at a smaller radius of curvature R_2 through a smaller outer surface of the wall element. However, since a smaller outer surface can be subjected to less of a load, the maximum energy flux density for large curvatures will be less than for small ones. From this fact, an important physical conclusion which applies to the construction of light arcs follows, namely, maximum thermal strength of a straight light arc is always greater than that for a toroidal one.

As we can see from Figure 5, for a fixed radius of curvature $j_{E\ max}$ can assume values that increase as the selected tube radius decreases. From Figure 6 we can see this state of affairs as seen from a physically qualitative standpoint. The smaller the tube radius, the wall thickness remaining the same, the larger the outer surface of the wall element under consideration for removal of the energy flux. This qualitative dependence of the maximum energy flux density on the tube radius is not specific for a torus. It is also found in the case of straight cylindrical arcs.

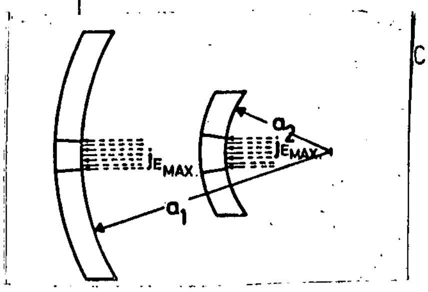

Figure 6. Diagram Showing the Dependence of the Maximum Energy Flux Density At $\varphi = \pi$ On the Tube Radius.

4. Conclusions /21

In view of its thermal strength, the toroidal arc with electrodes is less satisfactory than a straight arc. The situation remains the same if, as in the case of a cylindrical transpiration arc, an attempt is made to influence the radial heat loss through a radial influx of gas. A toroidal arc without electrodes can therefore be more advantageous than a cylindrical arc only if it is operated at a very high current. In this case, the energy losses at the electrodes of a cylindrical arc become so high that the toroidal arc without electrodes becomes more satisfactory despite the lower thermal strength.

I would like to thank Professor Schmitz and Dr. Druxes for numerous fruitful discussions.

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